Web Appendix to Components of bull and bear markets: bull corrections and bear rallies

John M. Maheu* Thomas H. McCurdy† Yong Song‡

1 Bull and Bear Dating Algorithms

Ex post sorting methods for classification of stock returns into bull and bear phases are called dating algorithms. Such algorithms attempt to use a sequence of rules to isolate patterns in the data. A popular algorithm is that used by Bry and Boschan (1971) to identify turning points of business cycles. Pagan and Sossounov (2003) adapted this algorithm to study the characteristics of bull/bear regimes in monthly stock prices. First a criterion for identifying potential peaks and troughs is applied; then censoring rules are used to impose minimum duration constraints on both phases and complete cycles. Finally, an exception to the rule for the minimum length of a phase is allowed to accommodate 'sharp movements' in stock prices.

There are alternative dating algorithms or filters for identifying turning points. For example, the Lunde and Timmermann (2004) (LT) algorithm identifies bull and bear markets using a cumulative return threshold of 20% to locate peaks and troughs moving forward. They define a binary market indicator variable I_t which takes the value 1 if the stock market is identified by their algorithm to be in a bull state at time t and 0 if it is in a bear state.

The classification of our data into bull and bear regimes using these two filters is found in Figure 1. The shaded portions under the cumulative return denote bull markets while the white portions of the figure are the bear markets. The exact dates for the bull and bear regimes can be found in Table 1. There are several features to note. First, the sorting of the data is broadly similar but with important differences. For example, during the 1930s the BB approach finds many more switches between market phases than does the LT algorithm. More recently, both identify 1987-12 as a trough but the subsequent bull phase ends in 1990-06 for LT but 2000-03 for BB. The average bear duration is similar (66 weeks) while the average bull

*Department of Economics, University of Toronto and RCEA, jmaheu@chass.utoronto.ca
†Rotman School of Management, University of Toronto & CIRANO, tmccurdy@rotman.utoronto.ca
‡CenSoC, University of Technology, Sydney, ysong1@gmail.com

duration is quite different, 117.0 weeks (BB) versus 166.7 (LT). In other words, the different parameters and assumptions in the filtering methods can result in a different classification of market phases.

Although the ex post dating algorithms can filter the data to locate different regimes, they cannot be used for forecasting or inference. In addition, since the sorting rule focuses on the first moment, it does not characterize the full distribution of returns. The latter is required if we wish to derive features of the regimes that are useful for measuring and forecasting risk. Also, as noted above, ex post dating algorithms sort returns into a particular regime with probability zero or one. However, the data provides more information allowing one to estimate probabilities associated with particular states.

Nevertheless, the dating algorithms are still very useful. For example, we use the LT algorithm to sort data simulated from our candidate parametric models in order to determine whether the latter can match commonly perceived features of bull and bear markets.

The Pagan and Sossounov (2003) adaptation of the Bry-Boschan (BB) algorithm can be summarized as follows:

1. Identify the peaks and troughs by using a window of 8 months.
2. Enforce alternation of phases by deleting the lower of adjacent peaks and the higher of adjacent troughs.
3. Eliminate phases less than 4 months unless changes exceed 20%.
4. Eliminate cycles less than 16 months.

Window width and phase duration constraints will depend on the particular series and will obviously be different for smoothed business cycle data than for stock prices. Pagan and Sossounov (2003) provide a detailed discussion of their choices for these constraints.

The Lunde and Timmermann (2004) dating algorithm defines a binary market indicator variable I_t which takes the value 1 if the stock market is in a bull state at time t and 0 if it is in a bear state. The stock price at the end of period t is labelled P_t. Our application of their dating algorithm can be summarized as: use a 6-month window to locate the initial local maximum or minimum.

Suppose we have a local maximum at time t_0, in which case we set $P_{t_0}^{\max} = P_{t_0}$.

1. Define stopping-time variables associated with a bull market as

$$\tau_{\max}(P_{t_0}^{\max}, t_0 \mid I_{t_0} = 1) = \inf\{t_0 + \tau : P_{t_0+\tau} \geq P_{t_0}^{\max}\}$$

$$\tau_{\min}(P_{t_0}^{\max}, t_0 \mid I_{t_0} = 1) = \inf\{t_0 + \tau : P_{t_0+\tau} \leq 0.8 P_{t_0}^{\max}\}$$

2. One of the following happens:

- If $\tau_{\max} < \tau_{\min}$, bull market continues, update the new peak value $P^{\max}_{t_0+\tau_{\max}} = P_{t_0+\tau_{\max}}$ and set $I_{t_0+1} = \cdots I_{t_0+\tau_{\max}} = 1$. Update $t_0 = t_0 + \tau_{\max}$ still as local maximum and continue with step 1 above.

- If $\tau_{\max} > \tau_{\min}$, we find a trough at time $t_0 + \tau_{\min}$ and we have been in a bear market from $t_0 + 1$ to $t_0 + \tau_{\min}$. Set $I_{t_0+1} = \cdots = I_{t_0+\tau_{\min}} = 0$. Record the value $P^{\min}_{t_0+\tau_{\min}} = P_{t_0+\tau_{\min}}$ and update $t_0 = t_0 + \tau_{\min}$ as local minimum. Go to step 3 below since t_0 is a local minimum now.

When t_0 is a local minimum:

3 Bear market stopping times are

$$\tau_{\min}(P^{\min}_{t_0}, t_0 \mid I_{t_0} = 0) = \inf\{t_0 + \tau : P_{t_0+\tau} \leq P^{\min}_{t_0}\}$$

$$\tau_{\max}(P^{\min}_{t_0}, t_0 \mid I_{t_0} = 0) = \inf\{t_0 + \tau : P_{t_0+\tau} \geq 1.2P^{\min}_{t_0}\}$$

4 One of the following happens:

- If $\tau_{\min} < \tau_{\max}$, bear market continues, update the new trough value, $P^{\min}_{t_0+\tau_{\min}} = P_{t_0+\tau_{\min}}$ and set $I_{t_0+1} = \cdots = I_{t_0+\tau_{\min}} = 0$. Update $t_0 = t_0 + \tau_{\min}$ and continue with step 3.

- If $\tau_{\min} > \tau_{\max}$ we find a peak at time $t_0+\tau_{\max}$ and we have been in a bull market from t_0+1 to $t_0+\tau_{\max}$. Set $I_{t_0+1} = \cdots = I_{t_0+\tau_{\min}} = 1$.Record the value $P^{\max}_{t_0+\tau_{\max}} = P_{t_0+\tau_{\max}}$ and update $t_0 = t_0 + \tau_{\max}$ as a local maximum. Go to 1 above since t_0 is a local maximum now.

This process is repeated until the last data point. All periods with $I_t = 1$ are in bull regime and $I_t = 0$ are in bear regime.

2 Results

Figure 2 displays the density of each of the 4 states of the MS-4. The differences in the illustrated densities are in accord with the parameter estimates. Differences in the spreads of the densities are most apparent but the locations are also different. There is no suggestion from these plots that states 1 and 2 are the same or that states 3 and 4 are the same, as a two-state Markov-switching model would assume.

Integrating state 1 and 2 gives the bear regime and doing the same for states 3 and 4 produces the bull regime. These densities are shown in Figure 3. The bear regime has a mean slightly below 0 but with a much larger variance than the bull regime. The implied unconditional density of returns is a mixture of these two regimes and displayed in the middle of the figure.

The Markov-switching models specify a latent variable that directs low frequency trends in the data. As such, the regime characteristics from the population model are not directly comparable to the dating algorithms of Section 1. Instead, we consider the dating algorithm as a lens to view both the S&P500 data and data simulated from our preferred MS-4 model. Using parameter draws from the Gibbs sampler, we simulate return data from the model and then apply the LT dating algorithm to those simulated returns. This is done many times and the average and 0.70 density intervals of these statistics are reported in Table 2 along with the statistics from the S&P500 data. Although our model provides a richer 4 state description of bull and bear markets it does account for all of the data statistics derived from the LT dating algorithm.

2.1 Identification of Historical Turning Points in the Market

2.1.1 1980-1985

In Figure 4, the market displays several moves between the bull market state and the bull correction state before a short-term move into a bear market in August of 1982. Once again the transition from a bull to bear market is through a bull correction state. However, the bear market that emerges has state 1 that lasts only about 4 weeks. This is followed by a bear rally that results in increased prices accompanied with substantial volatility. The bear rally turns into a bull market in late April of 1983, thereafter are periods of the bull market state and bull corrections.

References

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Table 1: BB and LT Dating Algorithm Turning Points

Troughs		Peaks		Troughs		Peaks	
BB[a]	LT[b]	BB	LT	BB	LT	BB	LT
1985-02					1940-06		1940-11
	1885-04			1942-05	1942-05	1943-07	
		1886-12		1943-12		1946-05	1946-05
1888-06		1890-06	1890-06	1948-02	1948-02	1948-06	
1890-12	1890-12	1892-03	1892-03	1949-06		1952-12	
1893-08	1893-08	1895-09	1895-09	1953-09		1956-07	1956-07
1896-08	1896-08	1897-09		1957-12	1957-12	1959-07	
1898-03		1899-04		1960-10			1961-12
1900-07		1902-09	1902-09	1962-07	1962-07	1966-02	1966-02
1903-10	1903-10		1906-01	1966-10	1966-10	1968-12	1968-12
		1906-10		1970-06	1970-06	1971-04	
1907-11	1907-11	1909-08	1909-08	1971-12		1973-01	1973-01
1910-08		1912-10		1974-10	1974-10	1976-09	
1914-12	1914-12	1916-11	1916-11	1978-03		1978-09	
1917-12	1917-12	1919-07	1919-07	1980-04		1980-11	1980-11
1921-06	1921-06	1929-09	1929-09	1982-08	1982-08	1983-06	
	1929-11		1930-04	1984-08		1987-08	1987-08
1932-06	1932-06		1932-09	1987-12	1987-12	1990-06	
	1933-03	1933-07	1933-07	1990-10		2000-03	2000-03
	1933-10		1934-02	2002-10	2002-10	2007-10	2007-10
1935-03	1935-03	1937-03	1937-03	2009-03	2009-03	2010-01	2010-01
1938-04	1938-04		1938-11				
	1939-04	1939-10	1939-10				

[a] BB: Bry and Boschan algorithm using Pagan and Sossounov parameters
[b] LT: Lunde and Timmermann algorithm

Table 2: Dating-algorithm filtering of data and simulated data

	S&P	MS-4
Avg. number of bears	29	31.7
		(22, 42)[a]
Avg. bear duration	63.1	55.9
		(40.5, 74.7)
Avg. bear amplitude[b]	-45.0	-43.4
		(-52.7, -35.8)
Avg. bear return	-0.71	-0.80
		(-1.08, -0.57)
Avg. bear std	3.16	3.15
		(2.60, 3.73)
Avg. number of bulls	28	31.4
		(22, 42)
Avg. bull duration	166.7	158.5
		(103.0, 235.3)
Avg. bull amplitude	66.4	60.2
		(46.3, 80.0)
Avg. bull return	0.40	0.39
		(0.31, 0.48)
Avg. bull std	2.53	2.42
		(1.97, 2.91)

[a] 70% density interval
[b] Aggregate return over one regime

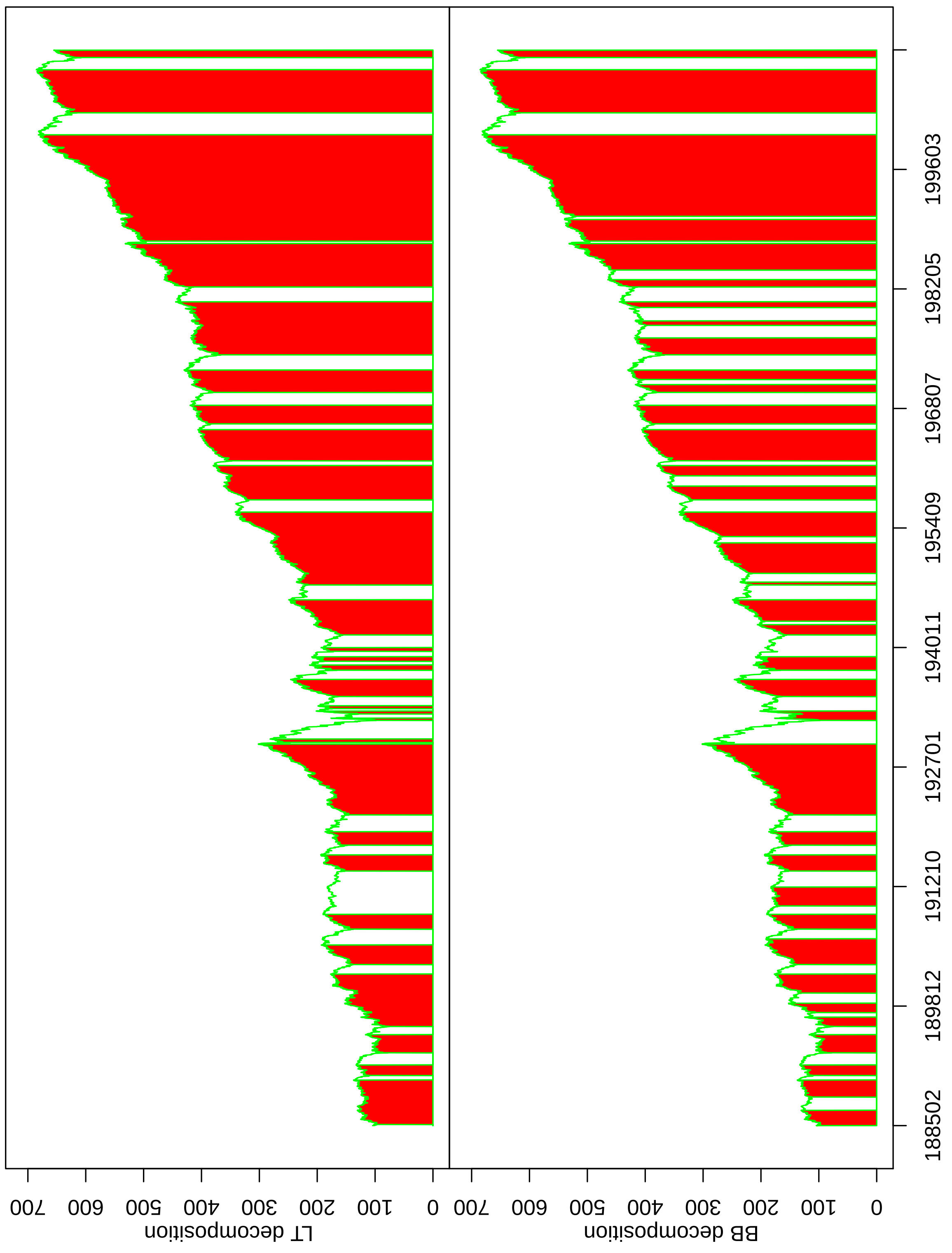

Figure 1: LT and BB dating algorithms

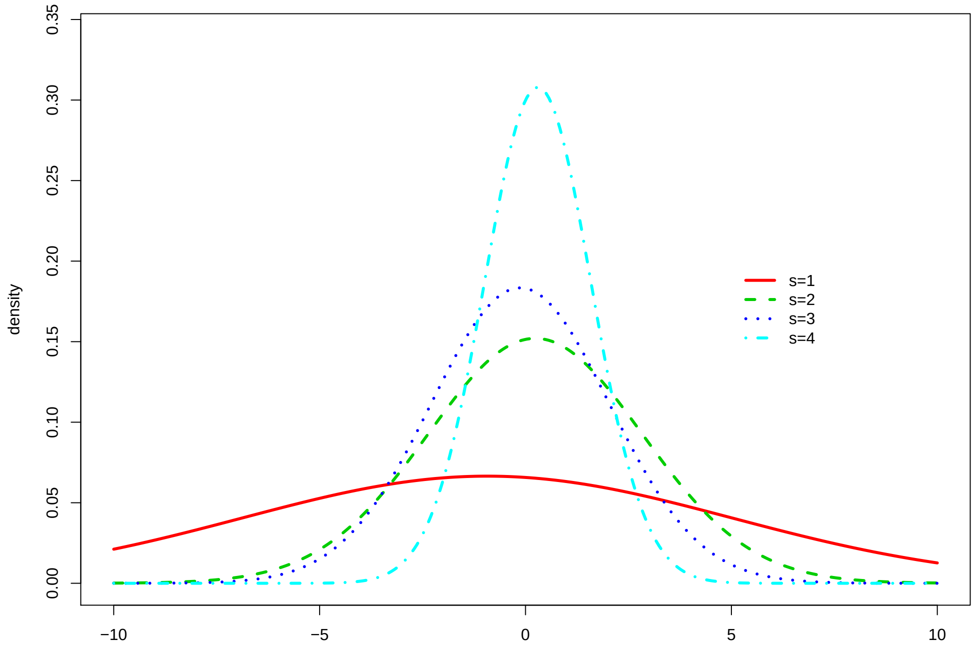

Figure 2: MS-4-States, State Densities

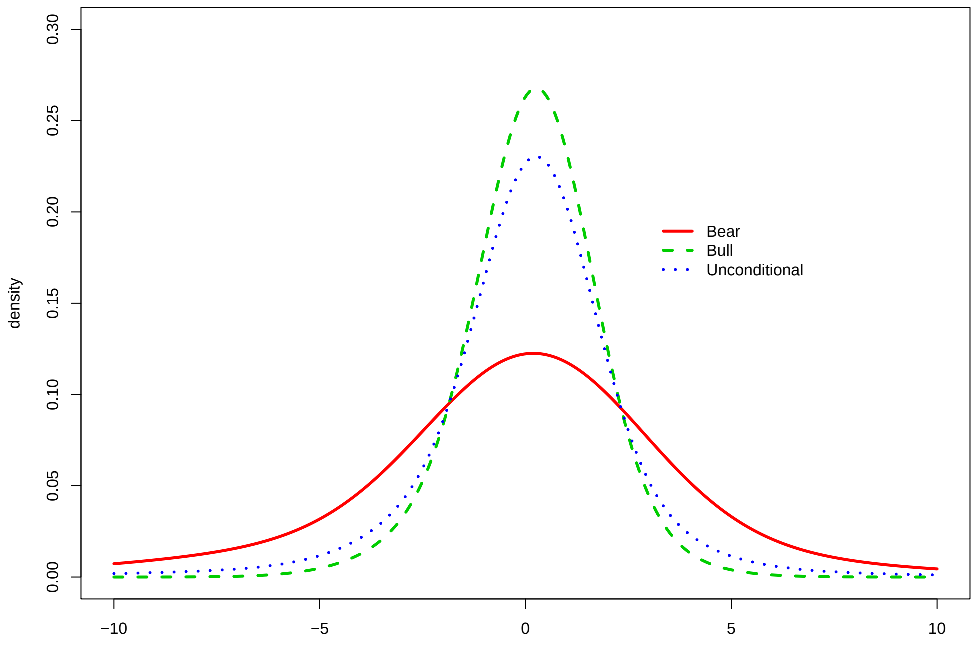

Figure 3: MS-4-States, Regime Densities

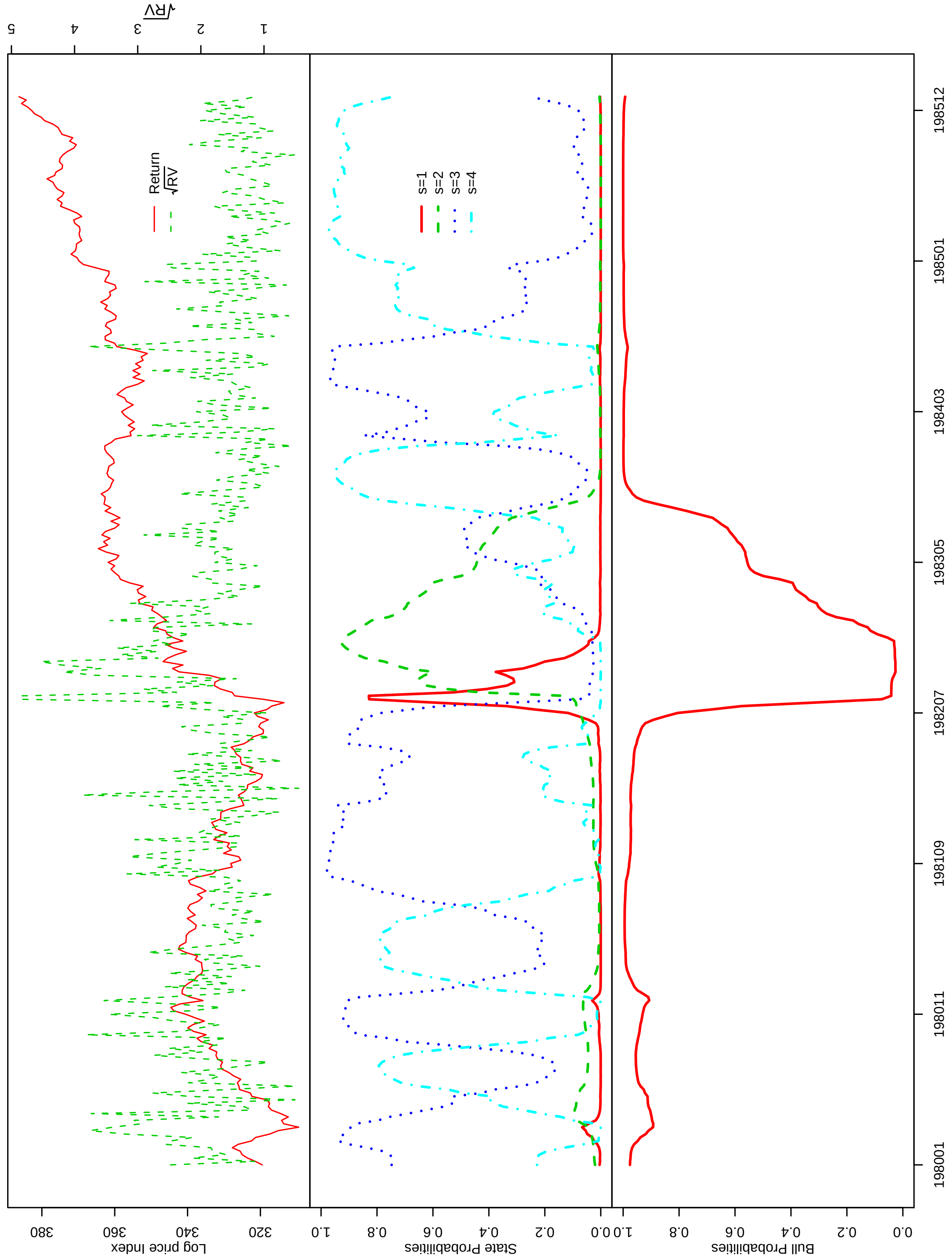

Figure 4: MS-4, 1980-1985